**Number Systems Introduction**

 Units **place**: 80 (which is 1)

 Eights **place**: 81 (which is 8)

 Sixty**-fours place**: 82 (which is 64)

 And so on...

**Decimal System (Base 10)**

**- Digits:** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

**- Example:** 345 (which means 3 hundreds, 4 tens, and 5 units)

**Binary System (Base 2)**

**- Digits:** 0, 1

**- Example:** 1011 (which means 1 eight, 0 fours, 1 two, and 1 one)

**Octal System (Base 8)**

**- Digits:** 0, 1, 2, 3, 4, 5, 6, 7

**- Example:** 57 (which means 5 eights and 7 units)

**Hexadecimal System (Base 16)**

**- Digits:** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A (10), B (11), C (12), D (13), E (14), F (15)

**- Example:** 2F (which means 2 sixteens and 15 units)

**Conversion**

**Decimal to Binary**

1. Divide the decimal number by 2.

2. Write down the remainder.

3. Repeat with the quotient until you reach 0.

4. The binary number is the remainders read from bottom to top.

**Example:** Convert 13 to binary.

* 13 ÷ 2 = 6 remainder 1
* 6 ÷ 2 = 3 remainder 0
* 3 ÷ 2 = 1 remainder 1
* 1 ÷ 2 = 0 remainder 1

So, 13 in binary is 1101.

**Example:** Convert 13.24 to binary.

* 13 ÷ 2 = 6 remainder 1
* 6 ÷ 2 = 3 remainder 0
* 3 ÷ 2 = 1 remainder 1
* 1 ÷ 2 = 0 remainder 1
* .24\*2 = 0.48, 0
* .48\*2 = 0.96, 0
* .96\*2 = 1.96, 1

So, 13 in binary is 1101.001

**Binary to Decimal**

1. Write down the binary number.

2. Starting from the right (least significant bit), multiply each bit by 2 raised to the power of its position.

3. Sum the results.

**Example:** Convert 1101 to decimal.

* 1 × 23 + 1 × 22 + 0 × 21 + 1 × 20
* 8 + 4 + 0 + 1 = 13

**Example:** Convert 101.101 to decimal.

* 1 × 22 + 0 × 21 +1 × 2-0 + 1× 2-1 + 0 × 2-2 + 1 × 2-3
* 4 + 0 + 1 + ½ + 0/4 + 1/8
* 5 + 5/8 =45/8 = 5.625

**Binary to Octal Conversion**

To convert from binary to octal, group the binary number in sets of three digits starting from the right (pad with zeros if necessary). Then, convert each group into its corresponding octal value.

0 (octal) = 000 (binary)

1 (octal) = 001 (binary)

2 (octal) = 010 (binary)

3 (octal) = 011 (binary)

4 (octal) = 100 (binary)

5 (octal) = 101 (binary)

6 (octal) = 110 (binary)

7 (octal) = 111 (binary)

**Steps:**

1. Group the binary digits in sets of 3, from right to left.
2. Convert each group into its octal equivalent.

**Example: Convert 101110 (binary) to octal.**

1. Group: 101 | 110
2. Convert:
   * 101 = 5 (octal)
   * 110 = 6 (octal)

So, 101110 (binary) = 56 (octal).

**Octal to Binary Conversion**

To convert from octal to binary, convert each octal digit into its 3-bit binary equivalent.

**Steps:**

1. Convert each octal digit into a 3-digit binary number.

Example: Convert 67 (octal) to binary.

1. Convert:
   * 6 = 110 (binary)
   * 7 = 111 (binary)

So, 67 (octal) = 110111 (binary).

**Hexadecimal to Binary Conversion**

Hexadecimal (or "hex") is a base-16 system, and each hex digit corresponds to a 4-bit binary number. To convert from hexadecimal to binary, follow these steps:

**Steps for Conversion:**

1. Write down the hexadecimal number.
2. Convert each hex digit to its 4-bit binary equivalent (see the table below).
3. Combine all the binary groups together in the same order.

**Hexadecimal to Binary Table:**

| **Hexadecimal** |  | **Binary** |
| --- | --- | --- |
| 0 |  | 0000 |
| 1 |  | 0001 |
| 2 |  | 0010 |
| 3 |  | 0011 |
| 4 |  | 0100 |
| 5 |  | 0101 |
| 6 |  | 0110 |
| 7 |  | 0111 |
| 8 |  | 1000 |
| 9 |  | 1001 |
| A (10) |  | 1010 |
| B (11) |  | 1011 |
| C (12) |  | 1100 |
| D (13) |  | 1101 |
| E (14) |  | 1110 |
| F (15) |  | 1111 |

**Example 1:** Convert 2F (hexadecimal) to binary.

1. Break the hex number into individual digits: 2 and F.
2. Convert each hex digit to binary:
   * 2 (hex) = 0010 (binary)
   * F (hex) = 1111 (binary)
3. Combine the binary groups:  
   2F (hex) = 00101111 (binary)

**Binary to Hexadecimal Conversion**

To convert binary to hexadecimal, group the binary number in sets of four digits starting from the right (pad with zeros if necessary). Then convert each group into its corresponding hexadecimal value.

**Steps:**

1. Group the binary digits in sets of 4 from right to left.
2. Convert each group into its hexadecimal equivalent.

**Example: Convert 110111 (binary) to hexadecimal.**

1. Group: 0001 | 1011
2. Convert:
   * 0001 = 1 (hexadecimal)
   * 1011 = B (hexadecimal)

So, 110111 (binary) = 1B (hexadecimal).

**Hexadecimal to Decimal Conversion**

To convert from hexadecimal to decimal, multiply each digit by 16 raised to the power of its position (starting from 0 on the right).

**Steps:**

1. Multiply each digit by 16 raised to the position (starting from 0).
2. Add the results.

**Example:** Convert 2F (hexadecimal) to decimal.

1. (2 × 16^1) + (F × 16^0) = (2 × 16) + (15 × 1) = 32 + 15 = 47

So, 2F (hexadecimal) = 47 (decimal).

**Decimal to Hexadecimal Conversion**

**To convert a decimal (base-10) number to hexadecimal (base-16), follow a process of repeatedly dividing the decimal number by 16 and keeping track of the remainders. Each remainder represents a hexadecimal digit.**

**Steps for Conversion:**

1. **Divide the decimal number by 16.**
2. **Write down the remainder. This is the first digit (rightmost) of the hexadecimal result.**
3. **Repeat the process with the quotient, dividing by 16 again.**
4. **Continue until the quotient becomes 0.**
5. **The remainders, read in reverse order, give the hexadecimal number.**

**Example 1: Convert 345 (decimal) to hexadecimal.**

1. **Divide by 16:  
   345 ÷ 16 = 21 quotient, 9 remainder  
   The remainder is 9 (the least significant digit of the hex number).**
2. **Divide the quotient (21) by 16:  
   21 ÷ 16 = 1 quotient, 5 remainder  
   The remainder is 5.**
3. **Divide the quotient (1) by 16:  
   1 ÷ 16 = 0 quotient, 1 remainder  
   The remainder is 1 (the most significant digit of the hex number).**

**So, reading the remainders in reverse order:  
345 (decimal) = 159 (hexadecimal)**

**Octal to Decimal Conversion**

To convert from octal to decimal, multiply each digit by 8 raised to the power of its position (starting from 0 on the right).

**Steps:**

1. Multiply each digit by 8 raised to the position (starting from 0).
2. Add the results**.**

**Example:** Convert 143 (octal) to decimal.

1. (1 × 8^2) + (4 × 8^1) + (3 × 8^0)  
   = (1 × 64) + (4 × 8) + (3 × 1)  
   = 64 + 32 + 3 = 99

So, 143 (octal) = 99 (decimal).

**Decimal to Hexadecimal Conversion**

To convert a decimal (base-10) number to hexadecimal (base-16), follow a process of repeatedly dividing the decimal number by 16 and keeping track of the remainders. Each remainder represents a hexadecimal digit.

**Steps for Conversion:**

1. Divide the decimal number by 16.
2. Write down the remainder. This is the first digit (rightmost) of the hexadecimal result.
3. Repeat the process with the quotient, dividing by 16 again.
4. Continue until the quotient becomes 0.
5. The remainders, read in reverse order, give the hexadecimal number.

**Example 1:** Convert 345 (decimal) to hexadecimal.

1. Divide by 16:  
   345 ÷ 16 = 21 quotient, 9 remainder  
   The remainder is 9 (the least significant digit of the hex number).
2. Divide the quotient (21) by 16:  
   21 ÷ 16 = 1 quotient, 5 remainder  
   The remainder is 5.
3. Divide the quotient (1) by 16:  
   1 ÷ 16 = 0 quotient, 1 remainder  
   The remainder is 1 (the most significant digit of the hex number).

So, reading the remainders in reverse order:  
345 (decimal) = 159 (hexadecimal)

**Decimal to Octal Conversion**

To convert decimal to octal, divide the decimal number by 8, record the remainder, and repeat the process with the quotient until the quotient is 0. The remainders (in reverse order) give the octal number**.**

**Steps:**

1. Divide the decimal number by 8.
2. Record the remainder.
3. Continue dividing the quotient by 8 until the quotient is 0.
4. The octal number is the remainders in reverse order.

**Example:** Convert 98 (decimal) to octal.

1. 98 ÷ 8 = 12 remainder 2
2. 12 ÷ 8 = 1 remainder 4
3. 1 ÷ 8 = 0 remainder 1

So, 98 (decimal) = 142 (octal).

**Octal to Decimal Conversion**

To convert an octal number (base-8) to a decimal number (base-10), follow these steps. In this conversion, each digit in the octal number is multiplied by a power of 8, based on its position (rightmost digit is at position 0).

**Steps for Conversion**

1. Write down the octal number

2. Starting from the **rightmost digit,** multiply each digit by \( 8^n \), where \( n \) is the position of the digit (starting from 0 on the right).

3. **Sum** all the resulting values.

**Example 1:** Convert `345` (octal) to decimal**.**

**1. Write the octal number:** 3458

**2. Multiply each digit by the power of 8 according to its position:**

8^2 = 3 \* 64 = 192

8^1 = 4 \* 8 = 32

8^0 = 5 \* 1 = 5

**3. Add the results:**

192 + 32 + 5 = 229

So, 3458 = 22910

**Arithmetic Operations**

**Addition**

**Binary Addition:**

- Similar to decimal addition but base 2.

- Carry over if the sum is 2 or more.

**Example:**

0+0 =0

0+1=1

1+1=0 (carry 1)

1+1+1=0 (carry 1)

And so on……...

**Example:** 1011

+ 1101

1 + 1 = 10 (0 carry 1)

1 + 0 + 1 (carry) = 10 (0 carry 1)

0 + 1 + 1 (carry) = 10 (0 carry 1)

1 + 1 (carry) = 10 (0 carry 1)

Final result: 11000

**Subtraction**

**Binary Subtraction:**

Similar to decimal subtraction but base 2.

Borrow if needed.

Example: 1101

- 1011

1 - 1 = 0

0 - 1 = -1 (borrow 1 from next digit: becomes 10 - 1 = 1)

( ~~1~~  = 0) = borrow 1

0 - 0 = 0

1 - 1 = 0

Final result: 0010 (or simply 10)

**Multiplication**

Binary Multiplication:

Similar to decimal multiplication but base 2.

Multiply and add as in decimal.

**Example:**  101 \* 11

101 × 1 = 101

101 × 10 = 1010

Add the results: 101 + 1010 = 1111

**Summary**

**Decimal:** Base 10

**Binary:** Base 2

**Octal:** Base 8

**Hexadecimal:** Base 16

Conversions involve dividing by the base and tracking remainders, while arithmetic operations follow the same principles but within the respective base system.